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Classification of quantum correlations in spacetime

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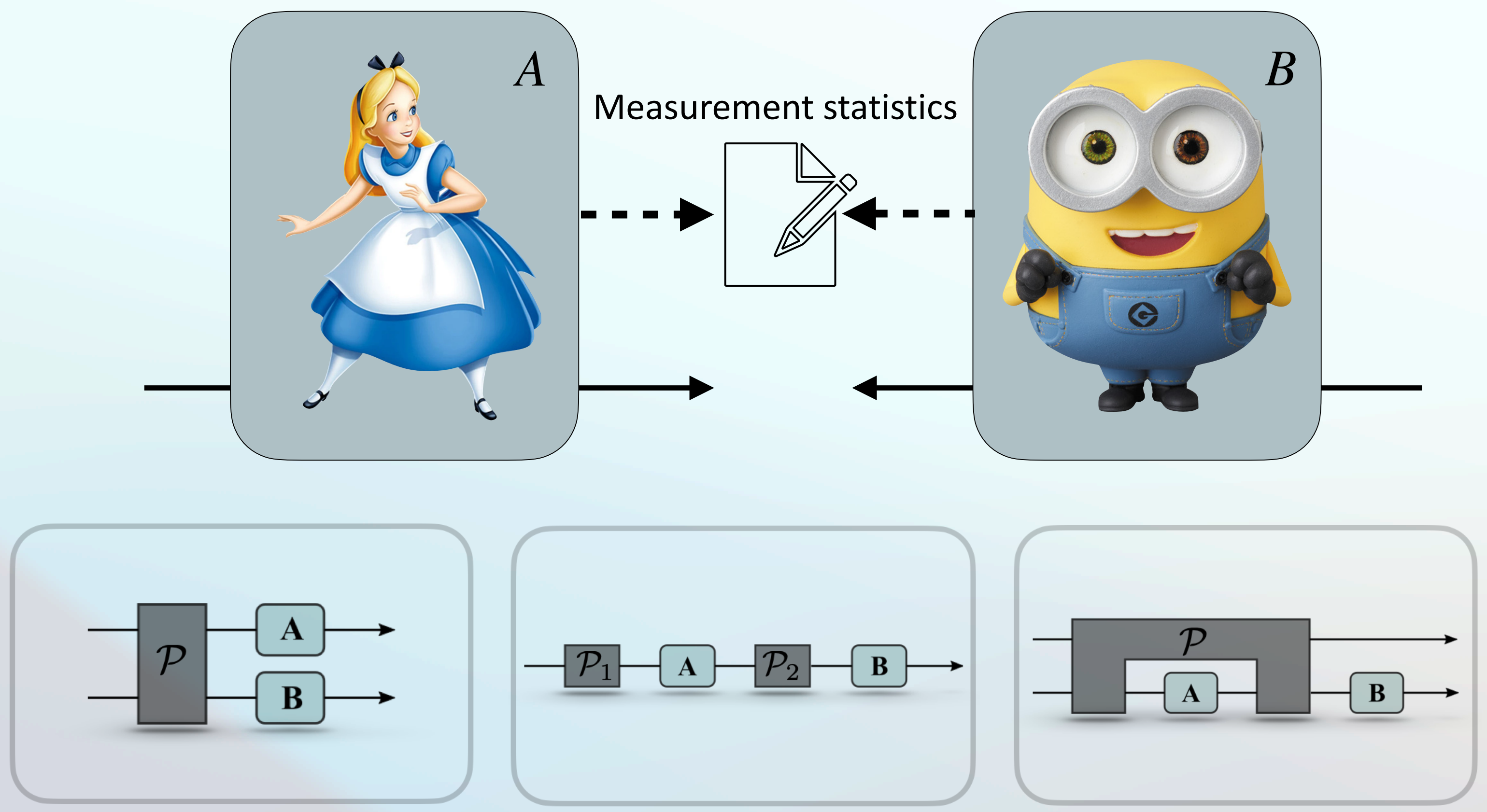
Abstract

The conventional representation of multipartite statistics in quantum information theory is through density operators, which implicitly treat the constituent subsystems as distinct degrees of freedom sharing the same temporal coordinate. The pseudo-density operator (PDO) representation generalizes this to admit causal structures with subsystems associated with the same degrees of freedom at distinct time instants.

- Result 1: we define the class $\mathscr{T}$ of PDO's compatible with a temporally distributed causal structure, and study its relation to the set $\mathscr{S}$ of density operators.
- Result 2: we provide a closed form expression of associated maps in terms of the given PDO, one of which would be a physical time evolution if the PDO belongs to, and we define a computable witness (``atemporality witness'') for non-membership in $\mathscr{T}$.
- Result 3: some entangled density operators are also in $\mathscr{T}$

Motivation

[Quantum observational scheme]



(a) Spatial distribution (b) Temporal distribution (c) Spatiotemporal distribution

FIG 1. Distributions of two quantum systems at distinct spacetime locations

With measurement statistics, can Alice and Bob tell which type the distribution of their systems is?

Framework: Pseudo-density operator (PDO) formalism
A PDO is a mathematical representation of measurement statistics

Result 1. Classification of pseudo-density operators

- A PDO is said to be *spatially consistent* if there are spatially distributed quantum systems that can give the same measurement statistics
- A PDO is said to be *temporally consistent* if there are temporally distributed quantum systems that give the same measurement statistics

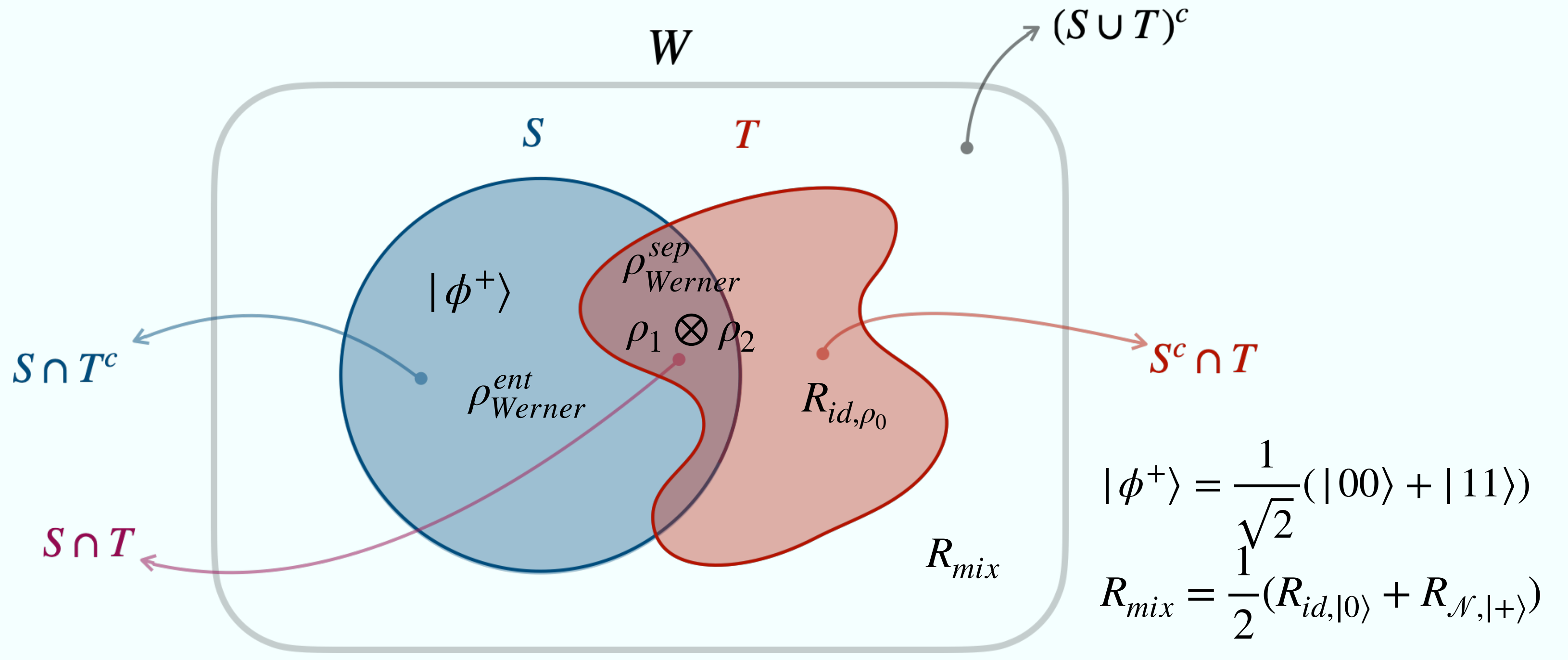


FIG 2. Venn diagram of the set of PDOs

$\mathscr{W}$: the set of PDOs
$\mathscr{S}$: the set of spatially consistent PDOs
$\mathscr{T} = \overrightarrow{\mathscr{T}} \cup \overleftarrow{\mathscr{T}}$: the set of temporally consistent PDOs,
$\overrightarrow{\mathscr{T}}(\overleftarrow{\mathscr{T}})$: the set of temporally forward (reverse) consistent PDOs

- $\mathscr{S} \cap \mathscr{T}^c$: **explainable strictly by spatially** distributed systems.
- $\mathscr{S}^c \cap \mathscr{T}$: **explainable strictly by temporally** distributed systems.
- $\mathscr{S} \cap \mathscr{T}$: **explainable both** by spatially distributed systems and temporally distributed systems.
- $(\mathscr{S} \cup \mathscr{T})^c$: **explainable neither** by spatially distributed systems nor temporally distributed systems.

Result 2. Atemporality witness

In what follows, we particularly investigate two qubit PDOs.

Lemma 1. For any PDOs R, there exists an associated forward map $\overrightarrow{\mathscr{M}}$ with R that satisfies $R = (id \otimes \overrightarrow{\mathscr{M}})K$, where $K \equiv \{\rho_0 \otimes \frac{I}{2}, S\}$, $\{A, B\} \equiv AB + BA$, S denotes the swap operator, and ρ_0 denotes the first marginal of R, and $R \in \overrightarrow{\mathscr{T}}$ iff there exists an associated map $\overrightarrow{\mathscr{M}}$ that is a physical time evolution.

Theorem 2. For any PDO R, there exists a systematic map-recovering method $\mathscr{X}$ that returns the set of the corresponding Choi operator of all associated forward map.

Definition [Choi atemporality].

A Choi atemporality f_{Choi} is defined as $f_{Choi}(R) \equiv \min\{\overrightarrow{f}_{Choi}(R), \overleftarrow{f}_{Choi}(R)\}$,

where the forward, reverse Choi atemporalities $\overrightarrow{f}_{Choi}$, $\overleftarrow{f}_{Choi}$ are given as

$$\overrightarrow{f}_{Choi}(R) \equiv \min_{\chi_{\overrightarrow{\mathscr{M}}} \in \mathscr{X}(R)} \frac{\|\chi_{\overrightarrow{\mathscr{M}}}\|_{tr} - 1}{2}, \quad \overleftarrow{f}_{Choi}(R) \equiv \min_{\chi_{\overleftarrow{\mathscr{M}}} \in \mathscr{X}(R')} \frac{\|\chi_{\overleftarrow{\mathscr{M}}}\|_{tr} - 1}{2},$$

respectively.

Note that R' denotes the swapped PDO, that is, $R' \equiv S(R)S^\dagger$.

Result 3. Atemporality and Entanglement

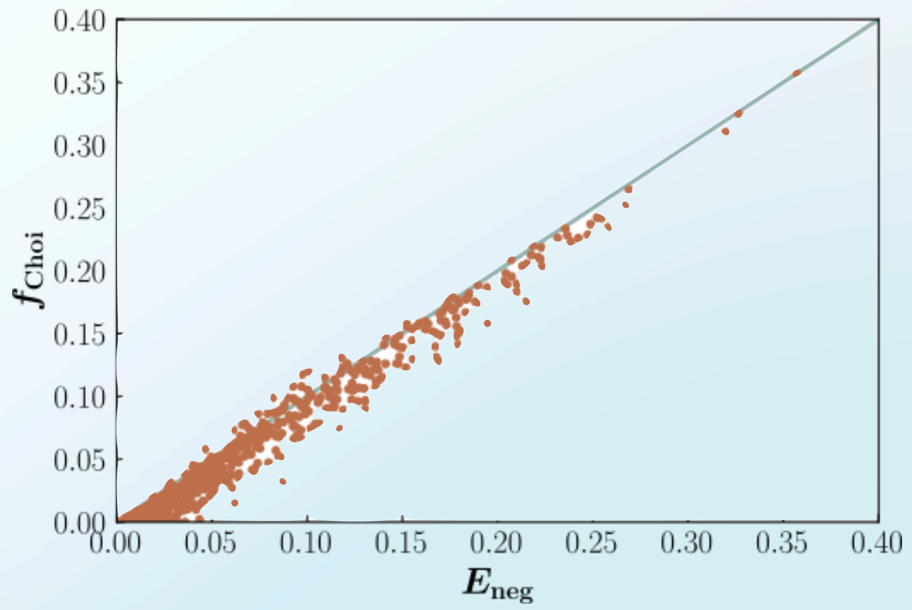


FIG 3. The graph of the entanglement negativity E_{neg} against the Choi atemporality, f_{Choi} for 1000 uniformly random PDO $R \in \mathscr{S}$.

Theorem 3. Let R be a density operator such that its marginals have full rank. Its Choi atemporality is equal to its entanglement negativity, $f_{Choi}(R) = E_{neg}(R)$ whenever R is pure or marginals of R are given by the maximally mixed state.

Theorem 4. Let R be a temporally consistent density operator. Then, its entanglement negativity $E_{neg}(R)$ is not greater than $\frac{\sqrt{2}-1}{2} \approx 0.2$.

Follow-up questions

Q1. If a density operator is separable, is it also temporally explainable?
Q2. Is a Choi atemporality a non-Markovianity monotone of processes?

References

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Supplementary

Pseudo-density operator formalism

Definition 1 (Pseudo-density operators)

An operator $\mathbf{R}$, defined as

$$\mathbf{R} \equiv \sum_{a=0}^{3} \sum_{b=0}^{3} \frac{\langle \boldsymbol{\sigma}_a, \boldsymbol{\sigma}_b \rangle}{4} \boldsymbol{\sigma}_a \otimes \boldsymbol{\sigma}_b. \quad (1)$$

is said to be a pseudo-density operator if $\langle \boldsymbol{\sigma}_a, \boldsymbol{\sigma}_b \rangle$ denotes the expectation value when Pauli measurements or no measurement are performed on quantum systems according to a, b.

Supplementary

Werner state

Definition 2 (Werner state)

Werner states are quantum states ρ_{AB} that satisfy $\rho_{AB} = (U \otimes U)\rho_{AB}(U^\dagger \otimes U^\dagger)\ \forall U$, where U denotes a unitary operator.

A two-qubit Werner state ρ_q is parameterized by $q \in [0,1]$ as $\rho_q = \frac{q}{3}P_s + (1-q)P_a$, where P_s, P_a are the projectors onto the symmetric space, the antisymmetric space, respectively. That is,

$$P_s = \frac{1}{2}(I \otimes I + S),$$

$$P_a = \frac{1}{2}(I \otimes I - S),$$

where $S = \frac{1}{2}\sum_{i=0}^{3} \sigma_i \otimes \sigma_i$ denotes a swap operator.

A two-qubit Werner state ρ_q is separable if and only if $q \geq 0.5$.

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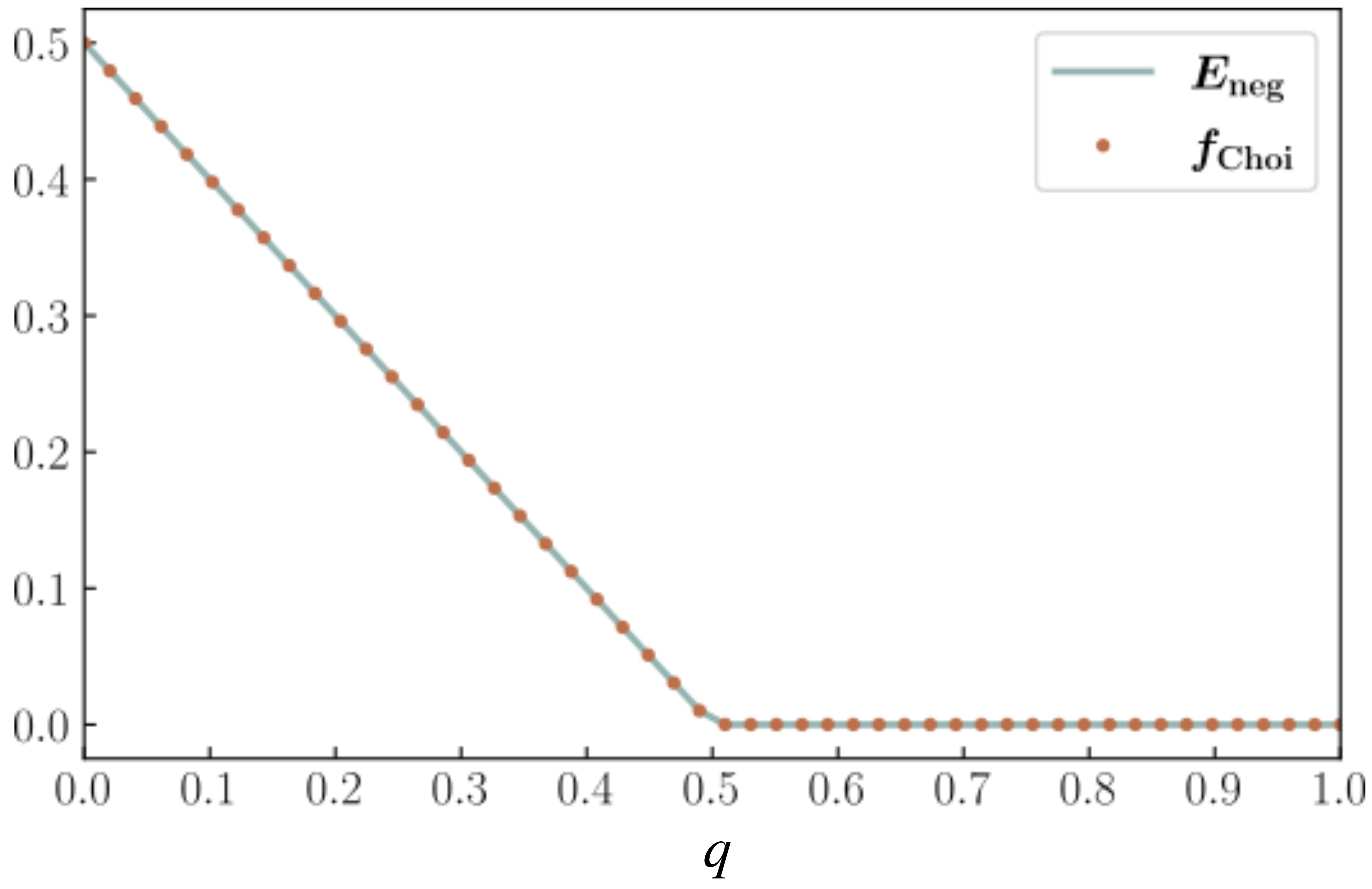

Supplementary

Map recovering method $\mathscr{X}$

Theorem 2. For any PDO $\mathbf{R}$, there exists a systematic map-recovering method $\mathcal{X}$ that returns the set of the corresponding Choi operator of all associated forward map with $\mathbf{R}$. In particular, if the associated initial state has full rank, the unique element $\mathbf{X}$ in $\mathcal{X}(\mathbf{R})$ is given by the following closed-form expression,

$$\mathcal{X}(\mathbf{R}) = \{\mathbf{X}\} \qquad \text{and} \qquad \mathbf{X} = (T \otimes \mathcal{I})\,(\mathbf{R} - \mathbf{S}_{\mathbf{R}})\,, \tag{50}$$

where T denotes a transpose map and

$$\mathbf{S}_{\mathbf{R}} \equiv \left(\boldsymbol{\rho}_0 - \frac{\mathbf{I}}{2}\right) \otimes \operatorname{tr}_A \left[\left(\frac{1}{2}\boldsymbol{\rho}_0^{-1} \otimes \mathbf{I}\right)\mathbf{R}\right] + \frac{\mathbf{I}}{2} \otimes \operatorname{tr}_A \left[\left((\mathbf{I} - \frac{1}{2}\boldsymbol{\rho}_0^{-1}) \otimes \mathbf{I}\right)\mathbf{R}\right], \tag{51}$$

where $\boldsymbol{\rho}_0 \equiv \operatorname{tr}_B \mathbf{R}$. Otherwise, $\mathcal{X}(\mathbf{R})$ is given by

$$\mathcal{X}(\mathbf{R}) = \{\mathbf{X}_{\mathcal{E}_{\boldsymbol{\tau}}} : \mathcal{E}_{\boldsymbol{\tau}} \text{ is an associated map with } \mathbf{R} \text{ and } \boldsymbol{\tau} \text{ is a quantum state}\}. \tag{52}$$