WHAT IS NONCLASSICAL ABOUT UNCERTAINTY RELATIONS?

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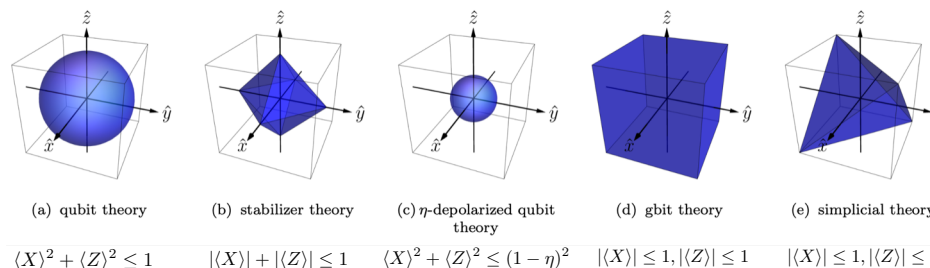
Motivation

- Uncertainty relations consist of bounds on the joint predictability of outcomes of distinct measurements on a single state.

- The existence of nontrivial uncertainty relations in quantum theory is generally considered to state a departure from the classical worldview.

- However, there exist theories that manifest uncertainty relations but also admit of a noncontextual ontological model.

- Which aspects of uncertainty relations, if any, witness contextuality?

Uncertainty relations

Definition

We consider uncertainty relations describing the tradeoff between the predictabilities of a pair of binary-outcome measurements (e.g., X and Z Paulis). Tradeoffs are in terms of sums of expectation values.

Examples

(a) qubit theory $\langle X \rangle^2 + \langle Z \rangle^2 \leq 1$

(b) stabilizer theory $|\langle X \rangle| + |\langle Z \rangle| \leq 1$

(c) η-depolarized qubit theory $\langle X \rangle^2 + \langle Z \rangle^2 \leq (1-\eta)^2$

(d) gbit theory $|\langle X \rangle| \leq 1, |\langle Z \rangle| \leq 1$

(e) simplicial theory $|\langle X \rangle| \leq 1, |\langle Z \rangle| \leq 1$

Noncontextual ontological models

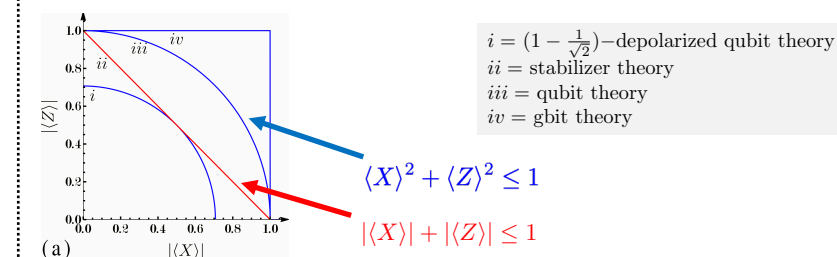### Ontological model of an operational theory

- Each system $\longrightarrow$ ontic state space Λ with ontic states $\lambda \in \Lambda$.

- Preparation P $\longleftrightarrow$ $\mu(\lambda|P)$

- Measurement and outcome M, y $\longleftrightarrow$ $\xi(y|M,\lambda)$

- Predicted statistics $\mathbb{P}(y|M,P) = \sum_{\lambda \in \Lambda} \xi(y|M,\lambda)\mu(\lambda|P)$

Noncontextuality

- Two preparations P, P' are *operationally equivalent*, $P \simeq P'$, if $\mathbb{P}(y|M,P) = \mathbb{P}(y|M,P')$ $\forall M$.

- In a preparation noncontextual ontological model, $P \simeq P' \implies \mu(\lambda|P) = \mu(\lambda|P')$.

Results

In any operational theory, if one can find a pair of measurements, X, Z, and a state that satisfies the condition of A_1^2-orbit-realizability, then noncontextuality implies that $|\langle X \rangle| + |\langle Z \rangle| \leq 1$.

$i = (1 - \frac{1}{\sqrt{2}})$−depolarized qubit theory
$ii =$ stabilizer theory
$iii =$ qubit theory
$iv =$ gbit theory

$\langle X \rangle^2 + \langle Z \rangle^2 \leq 1$

$|\langle X \rangle| + |\langle Z \rangle| \leq 1$

Condition of A_1^2-orbit-realizability

1. The state has equal predictability counterparts.

2. The state manifest operational equivalences with such counterparts.
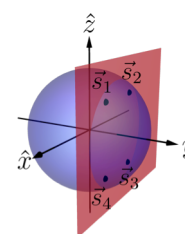
For theories where all states satisfy A_1^2-orbit-realizability our bound is a constraint on the form of the *ZX-uncertainty relation* within such theories.

Conclusion and future directions

o Given A_1^2-orbit-realizability, noncontextuality bounds the functional form of the ZX predictability tradeoff below a linear curve.

o The functional form of an uncertainty relation can witness contextuality.

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- What is nonclassical about interference phenomena?

References

[1] L. Catani, M. Leifer, G. Scala, D. Schmid and R.W. Spekkens arXiv:2207.11779 (2022).

[2]] L. Catani, M. Leifer, D. Schmid and R. Spekkens , 2111.137272 (2021).

[3] R.W. Spekkens, Phys. Rev. A **75**(3): 032110 (2007).

[4] R.W. Spekkens, Phys. Rev. A **71**: 052108 (2005).