Quantum Accelerated Causal Tomography: Circuit Considerations Towards Applications

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Introduction

AIM

To test hypothesis about the causal structure in a setting with different prior hypothesis of how we infer a set of variables that are causally related to each other.

Causal Hypothesis Testing (CHT)

Given variables and a set of hypothesis on causal relations among them as input, infer the correct hypothesis as output.

Quantum Speedup of CHT

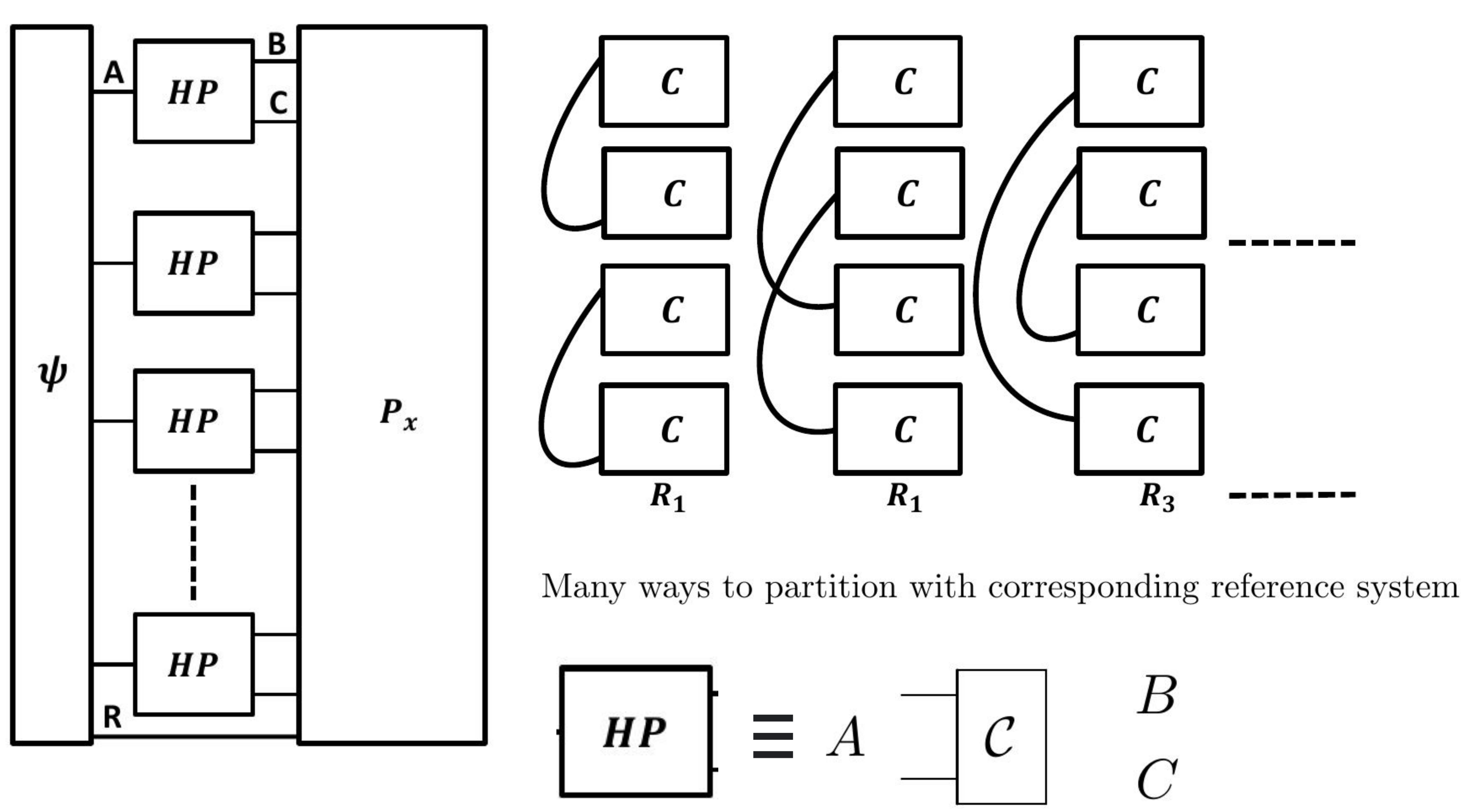
A causes B, C random uniform — A causes C, B random uniform

CLASSICAL ERROR PROBABILITY

A, B, C are random variables with same alphabet of size d < ∞. In parallel strategy where N input variables are initially set to some prescribed set of values the optimal error probability

$$P^{\mathrm{C}}_{\mathrm{err}} = \frac{1}{2d^{N-1}}$$

QUANTUM ERROR PROBABILITY



$$p_{\mathrm{err}} = \frac{r}{2d^N}\left(1-\sqrt{1-r^{-2}}\right) \xrightarrow{r>>1} \frac{1}{4rd^N}$$

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Pragmatic considerations

PROBLEM FORMULATION

Considering input variables i.e. the cause through set C and the output variables i.e. effects through E then

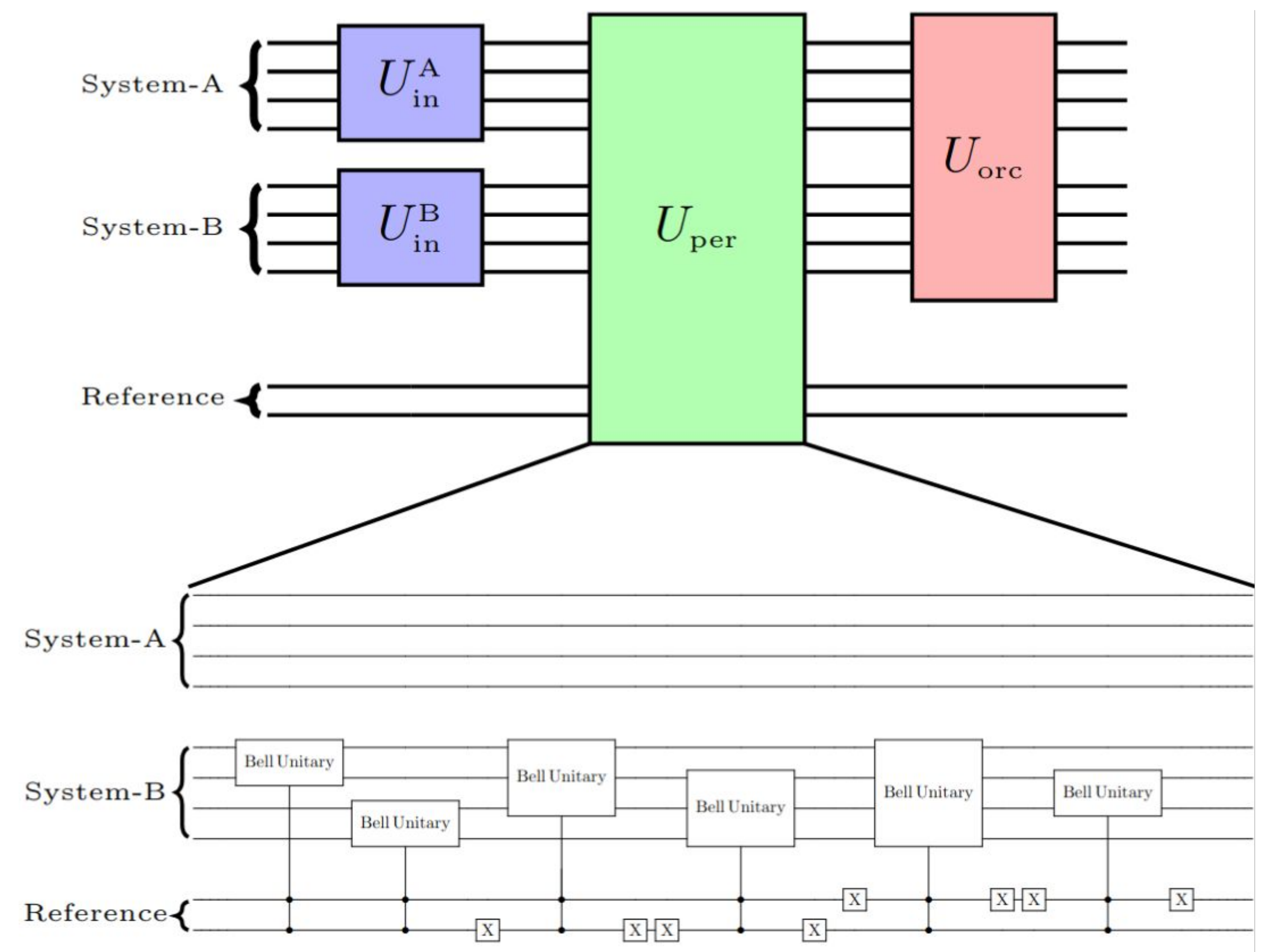
|C| = |E| = k = total number of causes/effects

also,

d = |c| = |e|

c, e are variables in C, E respectively.
The map from C to E is a **bijective function**.

MODEL IMPLEMENTATION



QUBIT COMPLEXITY AND ERROR PROBABILITY

The qubit requirement in model grows as

$$2N_A + \lceil \log(N_A) \rceil.$$

We introduce a correction factor proportional to the process distance between the two oracles.

$$p^{\mathrm{prac}}_{\mathrm{err}} = \frac{r}{2d^N}\left(1-\sqrt{1-r^{-2}}\right)\Delta\left[U_{\mathrm{orc}}, U^{\mathrm{alter}}_{\mathrm{orc}}\right] \xrightarrow{r>>1} \frac{\Delta\left[U_{\mathrm{orc}}, U^{\mathrm{alter}}_{\mathrm{orc}}\right]}{4rd^N},$$

Numerical results