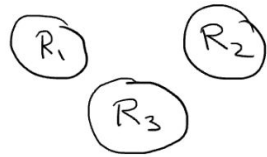


Causaoid Framework

for three regions

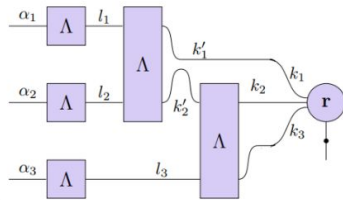


$$\Lambda = \{ \Lambda_{l_1}^{k_1}, \Lambda_{l_2}^{k_2}, \Lambda_{l_3}^{k_3}, \Lambda_{l_1, l_2}^{k_1, k_2}, \Lambda_{l_2, l_3}^{k_2, k_3}, \Lambda_{l_1, l_3}^{k_1, k_3}, \Lambda_{l_1, l_2, l_3}^{k_1, k_2, k_3} \}$$

where

- 1) 2<sup>nd</sup> rung of hierarchy
- 2)  $R_1 \bowtie R_2$   
 $R_2 \bowtie R_3$   
causally adjacent

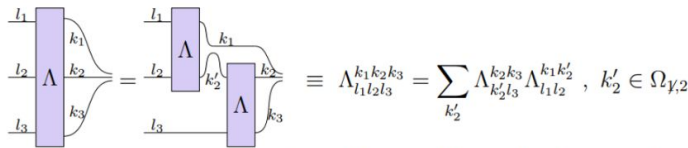
$$\Lambda = \{ \Lambda_{l_1}^{k_1}, \Lambda_{l_2}^{k_2}, \Lambda_{l_3}^{k_3}, \Lambda_{l_1, l_2}^{k_1, k_2}, \Lambda_{l_2, l_3}^{k_2, k_3} \}$$



$$l_i \in \Omega_i$$

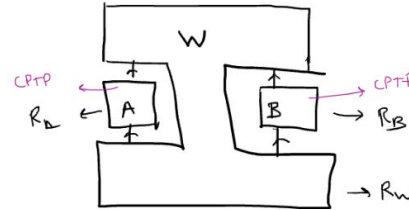
$$k_i, k_j \in \Omega_{ij}$$

$$\Omega_{ij} \subseteq \Omega_i \times \Omega_j$$



$$\text{where } \Omega_{1,2,3} = (\Omega_{1,2} \times \Omega_3) \cap (\Omega_1 \times \Omega_{2,3})$$

Process Matrix (Bipartite)



$$\Omega_A = \{ \mathbb{1}, G_i^{A_1}, G_i^{A_2} \}$$

$$\Omega_B = \{ \mathbb{1}, G_i^{B_1}, G_i^{B_2} \}$$

$$\Omega_W = \{ \mathbb{1}, G_i^{A_1}, G_i^{B_1}, G_i^{A_1} G_j^{B_1}, G_i^{A_1} G_j^{B_2}, G_i^{A_2} G_j^{B_1}, G_i^{A_2} G_j^{B_2} \}$$

$$\Omega_{AB} = \Omega_A \times \Omega_B$$

$$\Omega_{AW} = \{ \mathbb{1}, G_i^{B_1} \}$$

$$\Omega_{BW} = \{ \mathbb{1}, G_i^{A_1} \}$$

$$\Omega_{A \cup B, W} = \{ \mathbb{1} \}$$

$$\text{satisfies } \Omega_{A \cup B, W} = (\Omega_{AW} \times \Omega_B) \cap (\Omega_A \times \Omega_{BW})$$

**All Bipartite Process Matrices fall in the 2nd rung of the Hierarchy.  
(Quantum theory theory also belongs to the same rung)**

[1] Hardy, L. (2005). Probability theories with dynamic causal structure: a new framework for quantum gravity. *arXiv preprint gr-qc/0509120*.

[2] Sakharwade, N. (2022). An Operational Road towards Understanding Causal Indefiniteness within Post-Quantum Theories, UWSpace <http://hdl.handle.net/10012/17841>